Algebra 2, Quarter 4, Unit 4.1

Deriving Values of Special Angles on the Unit Circle and Graphing Trigonometric Functions

Overview

Number of instructional days: 12 (1 day = 45–60 minutes)

Content to be learned

- Construct the unit circle.
- Calculate radians.
- Solve Pythagorean identities.
- Graph trigonometric ratios (*intercept*; *intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity)*.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Apply the rules of special right triangles to the unit circle.
- Discover other patterns in the unit circle.

Model with mathematics.

- Create a unit circle on poster paper.
- Graph trigonometric functions.

Use appropriate tools strategically.

- Use graph paper for graphing trigonometric functions.
- Use calculators to perform basic calculations (should not be used for radians and rationalizing fractions).

Essential questions

- How does the radian measure relate to the arc length of the unit circle?
- How do the trigonometric values of cos, sin, and tan of the first quadrant of the unit circle relate to the other three quadrants?
- How is the Pythagorean Identity derived from the unit circle?
- How do the key components of a trigonometric function change the parent function graph?

Written Curriculum

Common Core State Standards for Mathematical Content

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

F-TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Model periodic phenomena with trigonometric functions

F-TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.⋆

Prove and apply trigonometric identities

F-TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

Interpreting Functions F-IF

Interpret functions that arise in applications in terms of the context

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*⋆

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does

this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In Kindergarten, students began to identify, describe, analyze, compare, create, and compose shapes. During grades 1 through 3, students developed reasoning with shapes and their attributes. In grade 4, students drew lines and angles and classified shapes by properties of their lines and angles. Graphing points on the coordinate plane and classifying two-dimensional figures into categories based on their properties occurred during grade 5. In the grade 7, students drew, constructed, and described geometrical figures and described the relationship between them. During grade 8, students learned and applied the Pythagorean Theorem. Previously in Geometry, students defined trigonometric ratios and solved problems involving right triangles including special right triangles. Students found the arc lengths of sectors of circles and understood and applied theorems about circles.

Current Learning

Students understand radian measure of an angle as the length of the arc on the unit circle. They explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. Student prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. For a function that models a relationship between two quantities, students interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Students choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Future Learning

Students will need the skills studied in this unit to extend the domain of trigonometric functions using the unit circle. Jobs that will require knowledge of the unit circle and Pythagorean Identities are insurance underwriters, funeral directors, and, of course, engineering.

Additional Findings

Mastery of the all the standards in this unit is important. All the standards are tested in the PARCC Assessments.

Algebra 2, Quarter 4, Unit 4.2

Interpreting Data and Making Inferences Using Data

Overview

Number of instructional days: 6 (1 day = 45–60 minutes)

Content to be learned

- Find the mean of data.
- Determine the standard deviation of data.
- Distribute a normal curve.
- Graph data on a scatterplot.
- Calculate function regression.
- Use random processes from experimentations.
- Make inferences from statistical models.
- Justify inferences made from statistical models.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Make and justify inferences about a data set. Another group will critique.
- Critique arguments made in articles with data sets.

Model with mathematics.

- Conduct an experiment and display data appropriately.
- Display a data set at least three different ways.

Look for and make use of structure.

- Determine the best display method for a data set.
- Determine the best method to conduct an experiment, a survey, etc.

Essential questions

- How do you determine the best model to collect data?
- Why should reports on data be critiqued?
- What process(s) should you use to make inferences and justify those inferences about data?
- How do you determine which regression to use on the calculator from the data?
- How do you determine the standard deviation of a data set?

Written Curriculum

Common Core State Standards for Mathematical Content

Interpreting Categorical and Quantitative Data★ S-ID

Summarize, represent, and interpret data on a single count or measurement variable

S-ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★

Summarize, represent, and interpret data on two categorical and quantitative variables [*Linear focus, discuss general principle*]

S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

b. Informally assess the fit of a function by plotting and analyzing residuals.

Making Inferences and Justifying Conclusions★ S-IC

Understand and evaluate random processes underlying statistical experiments

S-IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★

S-IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model*?★

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

S-IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★

S-IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★

S-IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.★

S-IC.6 Evaluate reports based on data.★

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In Kindergarten, students classified objects and counted the number of objects in categories. In grades 1 through 5, students represented and interpreted data. They did this by giving reasons why they grouped various objects, determining the reason for grouping, drawing various types of graphs to represent data, and interpreting graphs uses to represent data. In grade 6, students wrote equations using independent and dependent equations. Random samplings were made and information was gathered, and then inferences were made. Comparative inferences were made about two populations. Seventh-graders also investigated chance processes and developed, used, and evaluated probability models. In grade 8, students investigated patterns of association in vicariate data. In Algebra 1, students summarized, represented, and interpreted data on a single count or measurement variable. Students also summarized, represented, and interpreted data on two categorical and quantitative variables. Also in Algebra 1, students interpreted linear models.

Current Learning

Students use the mean and standard deviation to estimate populations, and they use the data set to graph them under the normal curve. Using the statistical process, students understand, evaluate, make inferences, and justify conclusions.

Future Learning

Students will use the content of this unit for the rest of their lives to interpret data displayed in reports on the job and to determine the validity of statistical data in newspaper and magazine articles. Students will need to understand the statistical process and the normal curve to make educated decisions in their lives, from advertising to educational testing. The content of this unit will be beneficial in a Statistics course.

Additional Findings

Students have already been tested on S-ID 4 and 6 in Algebra 1. Mastery of S-IC 1-6 needs to occur during Algebra 2. These standards are tested on the PARCC Assessment for Algebra II.

Algebra 2, Quarter 4, Unit 4.3

Applying Probability to Interpret Data

Overview

Number of instructional days: 8 (1 day = 45–60 minutes)

Content to be learned

- Compute theoretical and experimental probabilities for a sample space.
- Calculate the probability of an event.
- Using lists and tables, state the difference between independent and dependent events.
- Determine the union, intersection, and complement of events.
- Determine the complement and negation of a single event.
- Solve problems involving conditional probability.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Determine validity of results of probability experiments.
- Construct probability experiments and determine validity.

Model with mathematics.

- Give conclusions and data from probabilities in everyday language and everyday situations.
- Apply rules of probability to real-world situations.

Use appropriate tools strategically.

- Use graphing calculators to solve problems incorporating probabilities.
- Use manipulatives, such as coins or playing cards, to construct probability experiments.

Essential questions

- In what real-world situations would you encounter conditional probability?
- How does one determine whether events are independent or dependent?
- What is the relationship between theoretical and experimental probability?
- Why is the probability of dependent events different from the probability of independent events?

Written Curriculum

Common Core State Standards for Mathematical Content

Conditional Probability and the Rules of Probability★ S-CP

Understand independence and conditional probability and use them to interpret data

S-CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★

S-CP.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★

S-CP.3 Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B),* and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.★

S-CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★

S-CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*★

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In Kindergarten, students described and compared measureable attributes, and they classified objects and counted the number of objects in categories. In first grade, students organized, represented, and interpreted data with up to three categories. In second grade, students generated measurement data and drew a picture and bar graph to represent a data set with up to four categories. In third grade, students drew scaled picture graphs with several categories, and they generated measurement data and represented that data by marking a line plot with an appropriate scale. In fourth grade, students made a line plot to display a set of measurements in fractions of a unit. In fifth grade, students continued to work with line plots of measurements in fractions of a unit. In sixth grade, students developed an understanding of statistical variability and summarized and described distributions. In seventh grade, students used random sampling to draw inferences about a population, drew informal comparative inferences about two populations, and investigated chance processes. They also developed, used, and evaluated probability models. In eighth grade, students investigated patterns of association in bivariate data. In Algebra 1, students investigated chance processes, and they developed, used, and evaluated probability models. In Geometry, students understood independence and conditional probability and used them to interpret data, and they used the rules of probability to compute probabilities of compound events in a uniform probability model.

Current Learning

Students describe events as subsets of a sample space of outcomes or as unions, intersections, or complements of other events. Students understand that two events are independent if the probability of the events occurring together is the product of their probabilities. Students understand the conditional probability of A given B as P(A and B)/P(B) and interpret the independence of A and B. Students construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Students recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Future Learning

If moving on to statistics, students will calculate expected values and use them to solve problems and use probabilities to evaluate outcomes of decisions.

Additional Findings

All standards in this unit are to be tested by PARCC. This unit lends itself to projects and group work in conducting probability experiments.

Algebra 2, Quarter 4, Unit 4.4

Using Rules of Probability to Compute Models

Overview

Number of instructional days: 18 (1 day = 45–60 minutes)

Content to be learned

- Find the conditional probability of an event.
- Solve problems involving conditional probability.
- Determine the complement and the negation of a single event.
- Apply the addition and multiplication rules of probability.
- Use permutations and combinations to compute probabilities.

Mathematical practices to be integrated

Model with mathematics.

- Give conclusions and data from probabilities in everyday language and everyday situations.
- Apply rules of probability to real-world situations.

Look for and make use of structure.

- Use formulas to find probabilities in given problems.
- Look for patterns in data and apply findings to problems.

Look for and express regularity in repeated reasoning.

- Evaluate reasonableness of results in probability problems.
- Look for shortcuts that can be applied to probability problems.

Essential questions

- In what real-world situations would you encounter conditional probability?
- How does one determine whether events are independent or dependent?
- What is the relationship between theoretical and experimental probability?
- Why is the probability of dependent events different from the probability of independent events?
- How do the addition and multiplication rules affect probability outcomes?

Written Curriculum

Common Core State Standards for Mathematical Content

Conditional Probability and the Rules of Probability★ S-CP

Use the rules of probability to compute probabilities of compound events in a uniform probability model

S-CP.6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.★

S-CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.★

S-CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.★

S-CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.★

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In kindergarten, students described and compared measureable attributes, and they classified objects and counted the number of objects in categories. In first grade, students organized, represented, and interpreted data with up to three categories. In second grade, students generated measurement data and drew a picture and bar graph to represent a data set with up to four categories.

In third grade, students drew scaled picture graphs with several categories, and they generated measurement data and represented that data by marking a line plot with an appropriate scale. In fourth grade, students made a line plot to display a set of measurements in fractions of a unit. In fifth grade, students continued to work with line plots of measurements in fractions of a unit. In sixth grade, students developed an understanding of statistical variability and summarized and described distributions. In seventh grade, students used random sampling to draw inferences about a population, drew informal comparative inferences about two populations, and investigated chance processes. They also developed, used, and evaluated probability models. In eighth grade, students investigated patterns of association in bivariate data. In Algebra 1, students investigated chance processes, and they developed, used and evaluated probability models. In Geometry, students understood independence and conditional probability and used them to interpret data, and they used the rules of probability to compute probabilities of compound events in a uniform probability model.

Current Learning

Students compute and compare results of theoretical and experimental probability and independent and dependent events. Students find unions, intersections, and complements of events.

Future Learning

If moving on to Statistics, students will calculate expected values and use them to solve problems, and they will use probabilities to evaluate outcomes of decisions.

Additional Findings

All standards in this unit are to be tested by PARCC. This unit lends itself to projects and group work in conducting probability experiments.

Algebra 2, Quarter 4, Unit 4.5

Using Probability to Evaluate Outcomes

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Use probabilities to make fair decisions.
- Analyze decisions and strategies using probability concepts.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Offer solutions for scenarios.
- Justify solutions.

Model with mathematics.

- Graph or chart data set of probability.
- Design an experiment for probability.

Use appropriate tools strategically.

- Use a calculator with games to predict probability.
- Use number cubes to predict probability.

Essential questions

- What are some situations in which you can use probability?
- How does probability help with problems solving?

Written Curriculum

Common Core State Standards for Mathematical Content

Using Probability to Make Decisions⋆ S-MD

Use probability to evaluate outcomes of decisions

S-MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).⋆

S-MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).⋆

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

Statistics and probability began in sixth grade with the development of an understanding of statistical variability. Students also summarized and described distributions. The use of random sampling to draw inferences about a population began in seventh grade. The students also drew informal comparative inferences about two populations. They investigated chance processes and developed, used, and evaluated probability models. In eighth grade, students investigated patterns of association in bivariant data. In Algebra 1, students summarized, represented, and interpreted data on a single count and two categorical and quantitative variables. They also interpreted linear models.

Current Learning

Students use probability to evaluate outcomes of decisions.

Future Learning

Students will use the material learned in this unit to make informed decisions in probability situations for the rest of their lives. The information will provide background for a college statistics class. The occupations in which these skills will be useful are product testers, medical tester, and coaches (such as pulling players at the end of the game).

Additional Findings

The information in this unit lends itself to cross-curricular work and projects.